On $(1 + \varepsilon)$-approximate data reduction for the Rural Postman Problem*

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The Rural Postman Problem (RPP) is one of the most fundamental arc routing problems: Given an edge-weighted graph $G = (V, E)$ and a subset $R \subseteq E$ of required edges, the goal is to find a closed walk of minimum total edge-weight containing all edges of R. The problem is APX-hard and the best known approximation algorithm achieves an approximation factor of 3/2. Thus, in order to achieve $(1 + \varepsilon)$-approximations for RPP, generally exponential time is required and data reduction becomes crucial.

We study data reduction with provable performance guarantees, leading to a polynomial-size approximate kernelization scheme (PSAKS):

Theorem 1 ([1]) *Any RPP instance I can be reduced to an instance I' with $2b+O(c/\varepsilon)$ vertices in $O(n^3)$ time so that any α-approximate solution for I' gives an $\alpha(1+\varepsilon)$-approximate solution for I, for any $\alpha \geq 1$ and $\varepsilon > 0$, where b is the number of vertices incident to an odd number of edges of R and c is the number of connected components formed by the edges in R.*

The derived data reduction algorithm can in particular speed up expensive heuristics, since a close-to-optimal solution found by heuristics in I' will be close-to-optimal in I. It is also applicable to speed up algorithms for the more general Capacitated Arc Routing Problem that use RPP algorithms as subroutines.

We complement this positive result by showing that, under common complexity-theoretic assumptions, RPP cannot be reduced to solving instances of size $\text{poly}(c + b)$ if *optimal* solutions are required.

Theorem 2 ([1]) *RPP is WK[1]-hard parameterized by $c + b$.*

References

[1] R. van Bevern, T. Fluschnik, and O. Yu. Tsidulko, "On $(1 + \varepsilon)$-approximate data reduction for the Rural Postman Problem", arXiv:1812.10131, 2018.

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